Year 11 Mathematics Methods
Term 3

Investigation 2 *Take-home Section*
Exponential Functions and Sequences

Student Name: _____

There are no marks allocated to this take-home part of the Investigation.
It is not permitted for this part to be taken to the validation.

Introduction

Doctors are able to prescribe medication to treat people suffering from conditions that affect their ability to sleep, such as Parkinson's Disease. There are different kinds of medications that can be prescribed. An important requirement of any sleep medication is that their effect "wears-off" during the night. If the medication does not "wear off", the patient could experience drowsiness the next day and this could be dangerous, if, for example, they drive to work.

Consider the four medications listed below. For each one, the formula models the amount of medication in the patient's bloodstream at any time after taking a dose of the medication over the first 12 hours.

$$Zazolam\ Z = Z_0(0.84)^t$$
$$Nitrazepam\ N = N_0(0.92)^t$$
$$Pentobombitone\ P = P_0(1.08)^t$$
$$Methohexitone\ M = M_0(0.65)^t$$

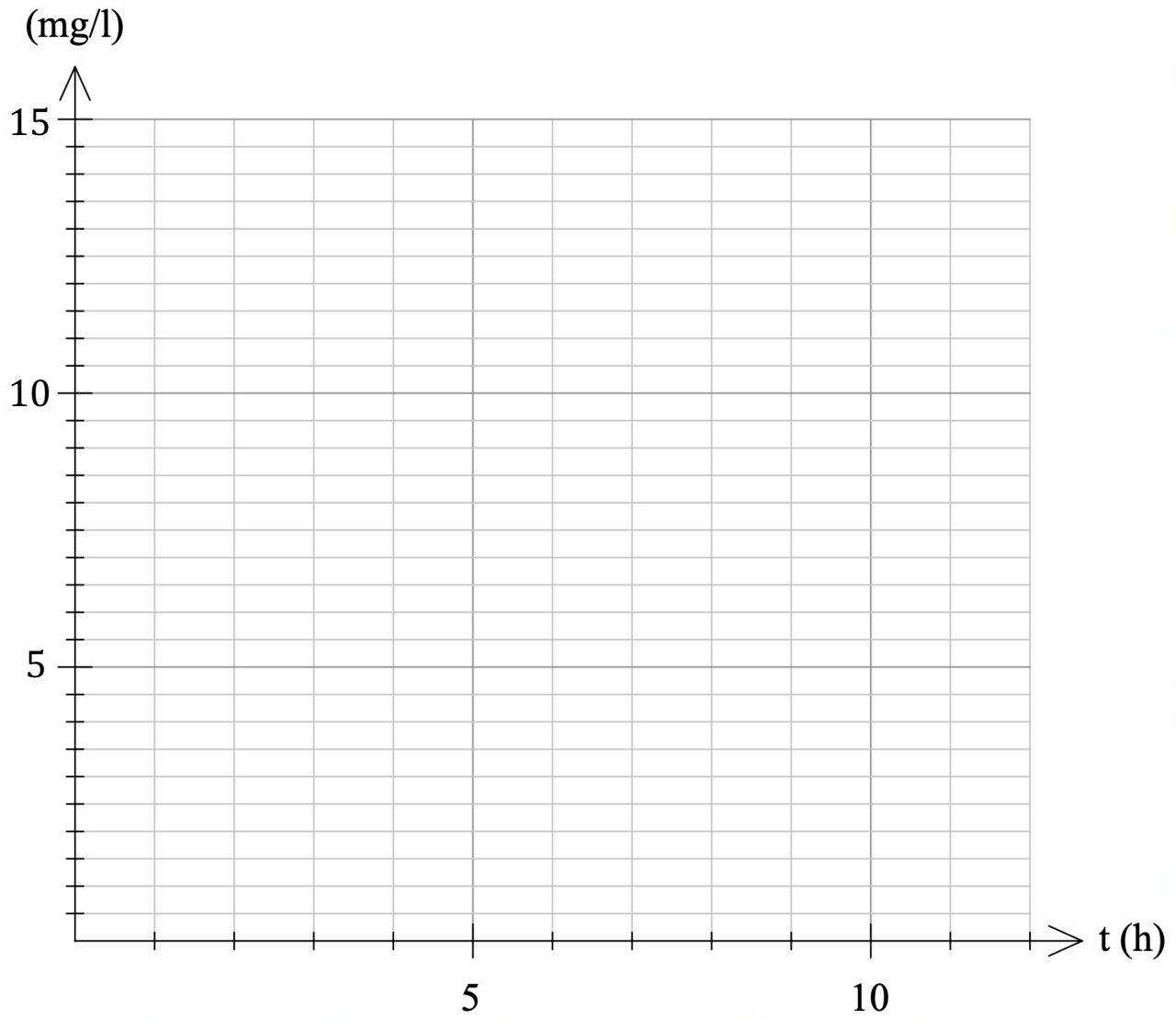
In these mathematical models above:

- t is the amount of time that has elapsed since taking the medication, measured in hours.
- *Z, N, P, M* is the amount of medication in the patient's bloodstream at any time, t , measured in *mg/l* (milligrams per litre).
- Z_0, N_0, P_0, M_0 relate to the size of the first dose taken by the patient, measured in *mg/l*.
    - For these models, we will assume the medication takes effect **immediately** after the first dose.

Task 1

On the set of axes below, graph the four mathematical models to compare how each of the medications will wear-off over a 12-hour period.

For all four medications, use a first dose of 10 mg/l. Ensure you label each mathematical model appropriately (a table of values may be useful when plotting key points).



Task 2

Only three of the models above could represent "real" medications.

a) (i) State which medication could not be "real".

- Which feature of the mathematical model enabled you to arrive at this conclusion?

- Which feature of the graph enabled you to arrive at this conclusion?

(ii) What would happen if you took the medication that could not be "real"?

b) For the three medications that could be considered "real":
   (i) state the rate of decay for each medication. Express the rate of decay as % per hour.

   (ii) state the amount of medication in the bloodstream at the end of the 12-hour period for each medication (to 4 decimal places).

c) If 10 mg/l *Methohexitone* was taken at 8pm at night, when would the amount of medication in the bloodstream be below 2 mg/l (to the nearest minute)?

Task 3

It is useful to consider the "half-life" of a medication. The half-life is the time it takes for the amount of medication in the bloodstream to halve (i.e. if we were to begin with 10mg/l of a medication, the half-life would be the amount of time it takes until we have 5 mg/l left).

a) Why is it not possible to determine the half-life of the medication that was considered "not real"?

b) Using your graph in Task 1, approximate the half-lives of the *Zazolam* and *Methohexitone* (to the nearest hour).

c) Calculate the half-life of *Nitrazepam* (to the nearest minute).

d) Does the half-life of these medications depend on the size of the initial dose? Support your answer with relevant calculations.

Would it be possible to write the mathematical models of these medications using our understanding of recursive sequences?